SAT-LCR-2-BFT(R)-03

MODELING OF MECHANICAL SYSTEMS BY THE REODYNAMIC METHOD

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Abstract: *There are many natural phenomena, in particular, technological processes that require fundamental researches. The basis of such researches usually are mathematical models based on the same type equations, most of which are differential equations of the second order or their systems. This is due to the possibilities of modern methods of analytical calculations and their visualization capabilities.*

The essence of many equations is the presence of a driving force and resistance forces of a different nature. According to these principles, the majority of mechanical, heat and mass, biological and other processes, and also economic analysis, are modeled. In order to quantify the interaction of mechanical systems with the working elements of technological equipment with the help of differential equations of motion of the second order, a number of mechanical processes were described, in particular, the movement of the knife in the product at cutting, the interaction of the product with the contact surface at the action of the forces of adhesion and friction, the interaction of complex visco-elastic systems with elements of equipment, in particular, transport and packaging systems, and others. The obtained mathematical dependencies allow to determine the displacement, speed and acceleration of the product at the contact with the elements of equipment, and energy indices – the work and the power for different laws of mutual movement.

Keywords: *Modelling, Reodynamic, Movation, Power, Differential, Equation.*

INTRODUCTION

There are many natural phenomena, in particular, technological processes that require fundamental researches. The basis of such researches usually are mathematical models based on the same type equations, most of which are differential equations of the second order or their systems. This is due to the possibilities of modern methods of analytical calculations and their visualization capabilities (Day L., Golding M., 2016).

Considered the mechanism of interaction between the working bodies of technological equipment - cam and other mechanisms with deformable bodies, which are compressed according to the laws characteristic for viscoelastic-plastic systems.

Existing methods of calculating transportation, packing, forming and other operations do not always take into account the complex rheological properties of products and the features of the working organs of the technological equipment, and their influence on the kinematic and energy characteristics of the process (Goots, V., Gubenia, O., Lukianenko, B. 2013).

The problem of the theoretical justification of the modes of deformation of the rheological system, the determination, according to the known laws of motion, of the cam mechanisms of energy characteristics and their optimization are being solved.

EXPOSITION

Reodynamic theory of deformation of viscoelastic systems. Rheology is the science of deformation and the flow of various materials that have properties that are excellent for classical (ideal) bodies. The main are flowing Newtonian fluids, elastic Guok and solid Sen-Venanian bodies (Qixin Zhong, Daubert C.R., 2013, Ahmed J., Ptaszek P., Basu S., 2017).

In a rheology, a stress is considered as the stimulus of the process, and deformation is a dimensionless relative value.

Rheology greatly expands, refines and at the same time complicates the theoretical regularities laid down (used) in the sciences, where only absolutely rigid or ideally fluid and elastic bodies are considered: the resistance of materials, theoretical mechanics, the theory of machines and mechanisms, the theory of elasticity, hydraulics and many others. In rheology, the force-stress (Pa · c), referred to the area unit, is considered as the motivator of the process, and the deformation is a dimensionless relative value [2].

Rheodynamic - an integral part of the rheology. Rheodynamic calculations are based on mathematical models of the interaction of deformable visco-elastic-plastic systems with working organs of equipment that can change their shape and deform with constant or variable tense and speed. Rheodynamic mathematical models allow to do the necessary technological calculations of machines and mechanisms, to determine the capacity, energy characteristics (deformation work, power) and to optimize them. The feature of rheodynamic is the possibility of research of material systems at rapidly changing of nonlinear stresses and speed deformation (Goots V., Gubenia O., Guts O., 2016).

In rheology, mathematical models are constructed taking into account different combinations of simple rheological bodies possessing one property: ideal elasticity (Guok body), ideal viscosity (Newtonian fluid), dry friction (Saint-Venant body). In rheodynamics, the characteristics of several single-type simple rheological bodies can be assumed to be the same. This approach greatly simplifies mathematical calculations in calculations. For example, the rigidity of all elastic elements entering into the rheological model (system) is assumed to be the same.

The basis for the analysis and construction of rheological models are mechanical models in the form of a spring, a damper, an object of friction connected in a certain sequence. In this case, the spring stiffness characteristic is assumed to be constant up to a certain amount of its compression, the damper's viscosity characteristic does not depend on the speed, and the dry friction of the friction body remains constant under any driving regimes and does not take into account the stagnation regime. In the work under the rheological system, two or more material objects interacting with one another and deformed as a result of stresses arising under the action of external driving forces are meant.

Examples of application of the theory. When designing the metering, transporting, packaging and other types of technological equipment, it is important to be able to calculate the force that occurs when deformation of the rheological systems of viscoelastic materials, when the modes of deformation according to different laws are set by the movement of the dosage-forming mechanism (Gavva, A. M, Khalaidzhi, V.V., Tokarchuk, S.V. 2011, James, S.J., James, C., Evans, J.A., 2006). Let's consider the case when the working bodies of the process equipment act on the rheological system, moving according to the predetermined modes of displacement x (t) according to the law, which is related to the design features of the cam mechanism. In this case, there are strains that need to be calculated (Parvini M., 2011).

Deformation (periodic compression) by the cam mechanism of the rheological system in the form of parallel-connected springs and a damper is represented schematically in Fig. 1.

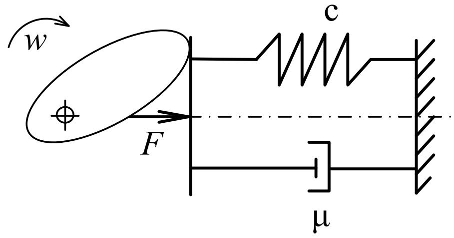

Fig. 1. Scheme of compression by a cam of a visco-elastic rheological system

Let us write down the law according to which the pusher of the cam mechanism moves, in this case a plate that is in contact with the rheological system and, accordingly, compresses the viscoelastic element

$$x(t) = R(1 - \cos wt)\,, \tag{1}$$

Where *R* – cam geometry; *w* – angular velocity of cam rotation (*w=const*); *t* – current cam rotation time, *x(t)* – the displacement of the cam follower, referred to the initial size of the rheological system.

An example of the relative displacement of the cam follower is shown graphically (fig. 2)

Having completed differentiated equation (1), we will find the compression rate. For our case, it is equal to the relaxation rate the return of the system to its original state.

$$\frac{dx(t)}{dt} = Rw\sin wt\,,\ \text{c}^{-1} \tag{2}$$

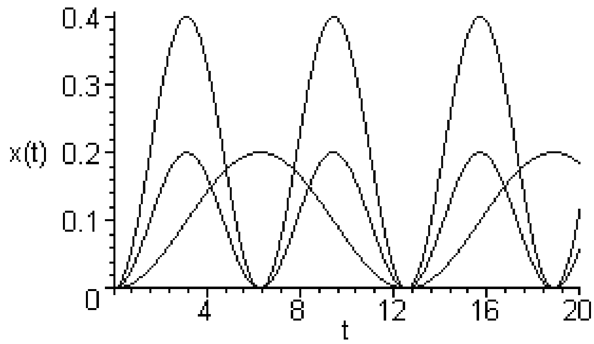

Fig.2. Graph *x(t)* for
(R=0,2; w_1=0,5c^{-1});
(R_1=0,1; w_1=0,5c^{-1});
(R_1=0,1; w=1c^{-1}).

A mathematical model of a viscoelastic system that is compressed under the action of a cam follower is equation [3]:

$$F = \mu\frac{dx(t)}{dt} + cx(t)\,, \tag{3}$$

Where *F* – deformation stress of a viscoelastic system, Pa.

We substitute in equation (3) offset *x(t)* and speed $\frac{dx(t)}{dt}$

$$F(t) = \mu Rw \sin wt + cR(1 - \cos wt)\,, \tag{4}$$

μ - viscous characteristic of the system, Pa·c;

c – elastic characteristic of the system, Pa.

To determine the energy characteristics of the deformation of a viscoelastic system, we find the rate at which the strain stress changes *F(t)*.

$$\frac{dF}{dt} = \mu Rw^2 \cos wt + cRw \sin wt \tag{5}$$

Knowing the magnitude of the rheological characteristics μ = 3000 Pa·c; c = 4000 Pa [1] and geometric parameters of the cam R = 0,2, having accepted the duration of work t from 0 to 50, and the angular velocity w from 0 to 0.4, we determine the strain stress $F(t,w)$.

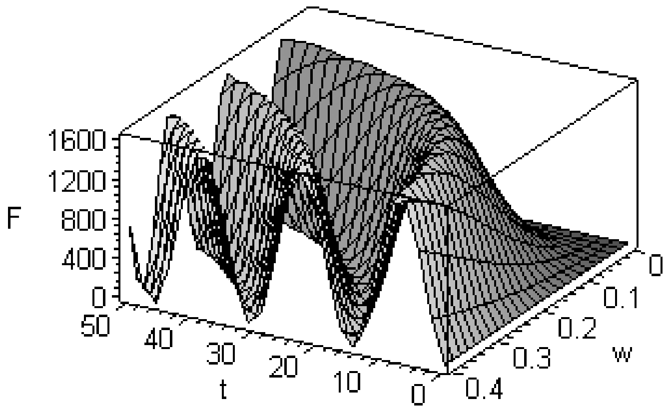

Fig. 3. Dependence of the stress of deformation of a viscoelastic system on the duration t and the angular velocity of rotation of the cam w.

In engineering when designing technological equipment, an important characteristic of a cam mechanism is the energy parameters of deformation by a pusher of a viscoelastic system.

Specific work of deformation (work per unit area of deformation) will be written [3].

$$A = \int_0^{t_1} F(t)dx(t) = \int_0^{t_1} \left[\mu Rw \sin wt + cR(1 - \cos t)\right](Rw \sin wt)dt =$$
$$\mu R + \frac{cR^2}{2} - \mu R \cos wt_1 - cR^2 \cos wt_1 + \frac{1}{2}cR^2(\cos wt_1)^2, \frac{Дж}{м^2} \tag{5}$$

By differentiating equation (5), we determine the energy expenditure (power), $\frac{W}{m^2}$

$$N = \frac{dA(t_1)}{dt_1} = \mu Rw \sin(wt_1) + cR^2 w \sin(wt_1) - cR^2 w \cos(wt_1)\sin(wt_1) \tag{6}$$

In t = 0..50 c; w = 0..0,4 c^{-1}; μ = 4000 Pa·c; c = 3000 Pa; R = 0,2 graph 3D dependence (5) is represented on fig. 3, and the dependence (6) on Fig. 4

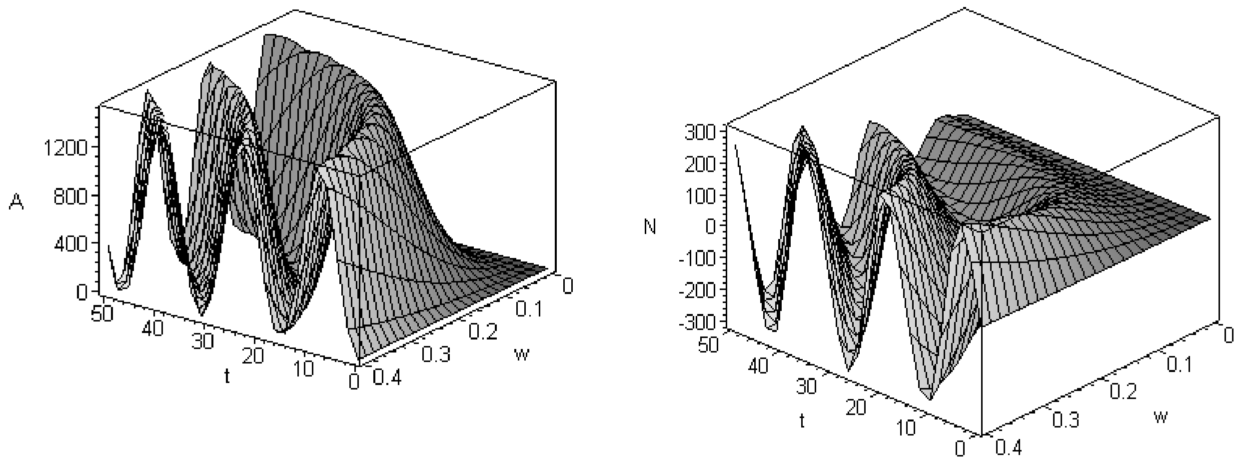

Fig. 4.

Total energy costs must take into account friction of the cam along the surface of the pusher. Then the complete deformation work A:

$$A = A_1 + A_2 \, , \tag{7}$$

where A_1 – specific work of frictional force;

A_2 – specific compression work.

CONCLUSION

The obtained mathematical dependencies allow to determine the displacement, speed and acceleration of the product at the contact with the elements of equipment, and energy indices – the work and the power for different laws of mutual movement.

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